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DISSIPATION DISCONTINUITIES

IN HYDROMAGNETIC SHOCK WAVES

by F. V. Coroniti

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ABSTRACT

Within the hydromagnetic approximation, the effects of resistive, viscous, and thermal conduction dissipation on the structure of shock waves is studied. A perturbation analysis about the upstream and downstream stationary points is developed, which, when coupled with the shock evolutionary conditions, determines the conditions for the formation of discontinuities in the shock structure. The viscous subshock for fast shock waves and the hydromagnetic analogue of the gas dynamic isothermal discontinuity for fast and slow shocks are analyzed. Very oblique fast shocks require both resistive and viscous dissipation for a steady shock structure. Strong slow shocks propagating nearly along the magnetic field fail to steepen if only resistive dissipation is included. The rotational discontinuity does not possess a stable shock structure for any of the dissipation processes considered.

1.0 Introduction

An initial approach in investigating the structure of hydromagnetic shock waves is to include the dissipative terms in the hydromagnetic fluid equations (Marshall, 1955; Anderson, 1963 and references therein; Leonard, 1966). With various approximations often including only one dissipation process, these equations reduce to a differential equation which describes the variation of the plasma quantities through the shock front and yields an estimate of the shock thickness. For strong shocks, however, a single dissipation process is frequently incapable of satisfying the Rankine-Hugoniot jump conditions. It is the purpose here to determine for what flow conditions multiple dissipation mechanisms are a required part of the shock structure.

The dissipation rate in a shock wave depends inversely on the shock thickness (Kantrowitz and Petschek, 1966). If the strengths of the dissipative terms are sufficiently different, i.e., if the characteristic scale length of each type of dissipation is different, the shock structure is often resolved into two or more regions: a broad shock transition in which some of the plasma quantities vary smoothly from their upstream to downstream values, and a thin or "discontinuous" region, called a subshock, located within the broad transition. Across the subshock some plasma quantities undergo a sharp transition, and remain roughly constant throughout the remainder of the shock front. The isothermal discontinuity (Landau and Liftshitz, 1959) for gas dynamic shocks and the viscous discontinuity for the fast perpendicular hydromagnetic shocks (Marshall, 1955) are examples of subshocks.

Considerable information about the shock structure is derivable from the evolutionary conditions (Anderson, 1963) and the propagation of linear waves with finite dissipation. Within the hydromagnetic approximation the dissipation processes considered are resistivity, viscosity, and thermal conductivity. For purposes of review and to fix notation, the evolutionary conditions are:

Fast Shock: $U_1 > C_{F_1}$; $C_{F_2} > U_2 \geq C_{I_2}$

Rotational Discontinuity: $C_{F_1} > U_1 > C_{I_1}$; $C_{I_2} > U_2 > C_{SL_2}$

Slow Shock: $C_{I_1} \geq U_1 > C_{SL_1}$; $C_{SL_2} > U_2$

C_F, C_I, C_{SL} are the fast, intermediate, and slow hydromagnetic speeds, respectively, and are defined in section 2.0; U is the normal component of the flow velocity; subscripts 1 (2) refer to upstream (downstream) flow conditions. Kantrowitz and Petschek (1966) demonstrated that the rotational discontinuity is non-evolutionary, a conclusion which is reinforced by the discussion of section 4.0.

A very brief and rough analysis of the effects of dissipation on linear hydromagnetic waves is given in section 2.0. Since the interest here is concerned with wave propagation near the shock front, the limit of wavelengths comparable with dissipation scale lengths is considered. A physical discussion of the downstream conditions leading to the formation of a subshock is given in section 3.0. From the steepening arguments and the linear wave analysis of section 2.0, the resistive fast shock propagating perpendicular to the magnetic field is shown to develop a subshock if the sound speed exceeds the flow velocity downstream. Oblique propagation of fast and slow shocks with resistive and thermal conduction dissipation is discussed using Friedrich's diagrams.

In section 4.0 a perturbation method is presented by which the stability, to be defined precisely below, of both the upstream and downstream stationary points of a shock for a given dissipation process is determined. The results of section 3.0 about the downstream point are readily recovered. Under certain conditions the upstream point of the resistive slow shock fails to steepen indicating that resistivity alone is incapable of starting the shock transition. Fast and slow shocks in which the magnetic field changes across the shock cannot be purely viscous, but are either resistive if the shock is weak or contain a combination of resistive and viscous dissipation. For the types of dissipation considered here the rotational discontinuity does not possess a stable transition from upstream to downstream flow conditions.

It is not the purpose here to determine for what Mach number and upstream plasma conditions multiple dissipation mechanisms are required. The criteria developed for subshock formation combined with the Rankine-Hugoniot relations will provide a solution to this problem. Since, except for particularly simple shock conditions, this would require a rather involved and tedious computation, only physical results will be emphasized here.

2.0 Linear Theory

2.1 Hydromagnetic Equations

The hydromagnetic equations with dissipation including Ohm's law and Maxwell's equations are (Landau and Lifshitz, 1960)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0$$

$$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v}\right) + \nabla\left(P + \frac{B^2}{8\pi}\right) - \frac{\underline{B} \cdot \nabla \underline{B}}{4\pi} = \eta \nabla^2 \underline{v} + \left(\zeta + \frac{\eta}{3}\right)\nabla(\nabla \cdot \underline{v})$$

$$\frac{\partial}{\partial t}\left(\frac{\rho v^2}{2} + \frac{B^2}{8\pi} + \frac{P}{\gamma - 1}\right) + \nabla \cdot \left(\rho \underline{v}\left(\frac{v^2}{2} + \frac{\gamma}{\gamma - 1}\,\frac{P}{\rho}\right) + \frac{\underline{B} \times \underline{v} \times \underline{B}}{4\pi}\right) \tag{2.1}$$

$$= \nabla \cdot \left(\frac{c^2}{(4\pi)^2 \sigma}\,\underline{B} \times (\nabla \times \underline{B}) + \underset{\approx}{\sigma} \cdot \underline{v} + \kappa \nabla T\right)$$

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{v} \times \underline{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \underline{B}$$

ρ is the mass density, $\underline{v}$ the fluid velocity, P the pressure assumed equal and isotropic for electrons and ions, $\underline{B}$ the magnetic field, T the temperature, γ the ratio of specific heats, and c the velocity of light. Gaussian units are used throughout. The adiabatic equation of state, P/ρ^γ = constant, was assumed in deriving the energy equation. The dissipation coefficients are the scalor conductivity σ, the thermal conductivity κ, and the two coefficients of viscosity η and ζ; $\underset{\approx}{\sigma}$ is the viscous stress tensor

$$\sigma_{ik} = \eta\left(\frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3}\,\delta_{ik}\,\frac{\partial v_\ell}{\partial x_\ell}\right) + \zeta\delta_{ik}\,\frac{\partial v_\ell}{\partial x_\ell} \tag{2.2}$$

δ_{ik} equals 1 if $i = k$, and zero otherwise.

For the purposes of this paper the dissipation coefficients are assumed constant, independent of the plasma state and position. As will be discussed below, this assumption is not overly restrictive. The transport properties described by these idealized dissipation coefficients can be interpreted as arising from particle-particle Coulomb collisions or, in a collisionless plasma, from the wave-particle interactions of plasma turbulence theory. For collisionless phenomena equations 2.1, however, are inappropriate and should be replaced with the two fluid equations for electrons and ions, including finite gyroradius effects and the contributions to the energy and momentum equations from turbulent wave fields. The hydromagnetic theory is useful, however, in elucidating the effects of various dissipation mechanisms; the conclusions, therefore, are expected to be appropriate in principle, if not in detail, to a collisionless theory.

2.2 Linear Hydromagnetic Waves

To obtain the linear hydromagnetic response, the plasma parameters in equations 2.1 are expanded about a uniform, stationary state, and second and higher order terms in the fluctuating quantities are neglected. In this section it is assumed that the stationary magnetic field is in the z direction, $\underline{B}_0 = B_0\,\hat{e}_z$, and that the stationary electric field and fluid velocity vanish. Stationary quantities are denoted by a zero subscript. After Fourier analyzing in space and time as

$e^{i\underline{k} \cdot \underline{x} - i\omega t}$, the above set of equations is reduced to the following dispersion relation

$$\left(\omega^2 - k^2\overline{C}_I^2 - \frac{i\eta\omega k^2 \sin^2\theta}{\rho_0}\right)\left[\omega^4 - \omega^2\left(k^2\overline{C}_A^2 + k^2\overline{C}_S^2\right) + k^4\overline{C}_A^2\overline{C}_S^2 \cos^2\theta \right.$$
$$\left. + i\,\frac{\omega\eta k^2}{\rho_0}\left(\omega^2 - k^2\overline{C}_S^2 - k^2\overline{C}_A^2\right)\right] = 0 \qquad (2.3)$$

where

$$\overline{C}_A^2 = \frac{C_A^2}{1 + \frac{ik^2C^2}{4\pi\omega\sigma}} - i\frac{\omega\eta}{\rho_0}$$

$$\overline{C}_S^2 = C_S^2\left[\frac{1 + i\,\frac{k^2\kappa}{\omega\rho_0}\,\frac{\gamma - 1}{\gamma}}{1 + i\,\frac{k^2\kappa}{\omega\rho_0}\,(\gamma - 1)}\right] - i\,\frac{\omega\eta}{\rho_0}\left(\zeta + \frac{\eta}{3}\right)$$

$$\overline{C}_I^2 = \overline{C}_A^2\cos^2\theta;\ C_A^2 = \frac{B_0^2}{4\pi\rho_0} \qquad (2.4)$$

$$C_S^2 = \frac{\gamma P_0}{\rho_0}$$

C_A is the Alfvén speed, C_S the sound speed, and θ is the angle between $\underline{k}$ and $\underline{B_0}$.

The dispersion relation 2.3 contains twelve modes, six of which are the hydromagnetic waves modified by dissipation, and the other six of which depend primarily on the dissipation and are highly damped (Baños, 1956). For the present purpose only a limited amount of information about the hydromagnetic waves is required, and the highly damped modes will be neglected. The derivation and discussion will be very imprecise, in particular with respect to comparing the real and imaginary parts of the frequency or wave vector (see Trehan, 1965, for a more complete discussion). The dissipative terms are assumed to be reasonably small so that a wave is only weakly damped. Since only the real part of the dispersion relation is needed below, the last term in the second bracket of 2.3 can be neglected without serious error, and 2.3 can be written as

$$\left(\frac{\omega^2}{k^2} - \overline{C}_I^2 - i\,\frac{\eta\omega \sin^2\theta}{\rho_0}\right)\left(\frac{\omega^2}{k^2} - \overline{C}_F^2\right)\left(\frac{\omega^2}{k^2} - \overline{C}_{SL}^2\right) = 0 \tag{2.5}$$

where

$$\left.\begin{matrix}\overline{C}_F^2 \\ \overline{C}_{SL}^2\end{matrix}\right\} = \frac{\overline{C}_A^2 + \overline{C}_S^2}{2} \pm \left(\left(\frac{\overline{C}_A^2 + \overline{C}_S^2}{2}\right)^2 - \overline{C}_S^2\overline{C}_A^2 \cos^2\theta\right)^{1/2} \tag{2.6}$$

If the dissipative terms are dropped from $\overline{C}_A$, $\overline{C}_S$, 2.4 defines the intermediate speed, C_I, and 2.6 the fast, C_F, and slow, C_{SL}, hydromagnetic speeds (Kantrowitz and Petschek, 1966).

Before proceeding it is convenient to define three subsidiary dissipation or diffusion lengths:

$$r_m = \frac{c^2}{4\pi\sigma\, C_{HM}} \tag{2.7}$$

$$r_e = \frac{\left(\frac{4}{3}\eta + \zeta\right)}{\rho_0\, C_{HM}} \tag{2.8}$$

$$r_t = \frac{\kappa}{\rho_0\, C_{HM}} \tag{2.9}$$

C_{HM} is to be assigned the hydromagnetic speed of the particular wave being considered; r_m and r_e are the lengths that make the magnetic Reynolds number and the viscous Reynolds number equal to unity; r_t is the scale length for thermal diffusion.

The discussion of subshocks in section 3.0, involves consideration of hydromagnetic waves or wave packets localized to the region of the shock layer (Anderson, 1963). Since the lengths defined by 2.7, 2.8, and 2.9 are typical of the shock layer thickness formed by their respective dissipation processes, the appropriate linear waves have $kr_m >> 1$, $kr_e >> 1$, and $kr_t >> 1$. These three scale lengths are usually of different magnitude, thus permitting the effects of each on the hydromagnetic wave speeds to be considered separately. Only the phase velocity of the waves is needed, so that ω and k are now considered real, and imaginary contributions to the frequency or wave vector are dropped.

2.2.1 Resistivity

When resistivity is the most important dissipation process, only the Alfvén speed is affected so that $\overline{C}_A^2$ becomes

$$\overline{C}_A^2 = \frac{C_A^2}{1 + k^2 r_m^2} \tag{2.10}$$

If the wavelength is small compared to the resistive diffusion length, $kr_m >> 1$, the effective Alfvén speed in the plasma is reduced to zero. In these waves the perturbation currents are resistively dissipated so that magnetic fluctuations do not propagate. Setting $\overline{C}_A^2 = 0$ in 2.4

and 2.6, the three hydromagnetic speeds become $\bar{C}_I = \bar{C}_{SL} = 0$ and $\bar{C}_F = C_S$; the intermediate and slow waves cease to propagate, and the fast wave becomes an isotropic sound wave.

2.2.2 Thermal Conductivity

If $\sigma = \infty$ and $\eta = \zeta = 0$, the sound speed becomes

$$\bar{C}_S^2 = \frac{\gamma P_0}{\rho_0} \frac{1 + k^2 r_t^2 \dfrac{(\gamma - 1)^2}{\gamma}}{1 + k^2 r_t^2 (\gamma - 1)^2} \tag{2.11}$$

When $kr_t \gg 1$, $\bar{C}_S^2 \rightarrow \tilde{C}_S^2 = P_0/\rho_0$; the effective ratio of specific heats becomes unity, which is characteristic of an isothermal plasma. For these waves heat is diffused sufficiently rapidly that no temperature fluctuations are propagated. The intermediate speed is unaffected for $kr_t \gg 1$, and substituting $\tilde{C}_s$ for $\bar{C}_s$ in 2.6 defines the fast and slow speeds $\tilde{C}_F$ and $\tilde{C}_{SL}$. Note that $C_F > \tilde{C}_F$ and $C_{SL} > \tilde{C}_{SL}$.

2.2.3 Viscosity

Setting $\sigma = \infty$ and $\kappa = 0$, the dispersion relation 2.3, after a bit of manipulation, becomes approximately

$$\frac{\omega^2}{k^2} = \frac{C_I^2}{1 + k^2 r_e^2}$$

$$\frac{\omega^2}{k^2} = \frac{C_F^2}{1 + k^2 r_e^2 \dfrac{C_F^2}{C_A^2 + C_S^2}} \tag{2.12}$$

$$\frac{\omega^2}{k^2} = \frac{C_{SL}^2}{1 + k^2 r_e^2 \dfrac{C_{SL}^2}{C_A^2 + C_S^2}}$$

For $kr_e >> 1$, the phase velocities of all three hydromagnetic waves are reduced to zero. Hence, as might be expected, in a very viscous plasma no fluctuations propagate.

In summary for kr_m large, no magnetic information is transmitted leaving only pressure fluctuations propagated by the fast wave isotropically at the sound speed. Waves for which kr_t is large propagate isothermally in the plasma. The fast and slow wave speeds are somewhat reduced. Finally for kr_e large, viscosity slows all three hydromagnetic wave speeds to zero. Although the derivation given here of the dissipative effects on the linear hydromagnetic waves has been extremely rough, the conclusions are essentially correct. In the next section this information permits a physical discussion of the effects of different dissipation processes on the shock structure.

3.0 Physical Discussion of Downstream Subshocks

To understand the physical basis for the formation of a subshock, a slightly elaborated form of an argument due to Kantrowitz and Petschek (1966) is presented in section 3.1 for the particularly simple case of a perpendicular resistive fast shock. After establishing the fundamental concepts, oblique progagation for fast and slow shocks in which resistivity and thermal conductivity separately provide the shock dissipation is discussed in section 3.2.

3.1 Perpendicular Fast Shock

Consider the problem in which a piston launches a fast non-linear pulse perpendicular to a uniform magnetic field. Following the arguments reviewed by Kantrowitz and Petschek (1966), the pulse steepens until its thickness reaches the longest dissipation length consistent with the entropy production necessary to satisfy the Rankine-Hugoniot conditions. Assume a steady shock is thereby formed in which resistivity provides all the dissipation so that the shock thickness is the order of the magnetic diffusion length.

The piston now launches another fast wave which, by the evolutionary conditions, must catch up with the shock front. This wave steepens until its gradients are also the order of the magnetic diffusion length, and from section 2.2.1 its propagation speed is then reduced to the sound speed. If the flow velocity behind the shock exceeds the sound speed, the wave cannot reach the shock and the shock structure remains steady; on the other hand, if the flow velocity behind is less than the sound speed, the wave overtakes the shock thus providing the shock with additional energy. Therefore, since resistivity cannot prevent the shock strength from increasing, the shock front continues to steepen until the next smaller dissipation length, for example, viscosity, is reached.

The waves now catching up with the shock must steepen until their wave length becomes comparable to the viscous scale length. However from 2.2.3, viscosity reduces the fast wave speed to zero. Since the flow velocity change across the shock can be reduced by at most a factor of about four, the fluid carries all further waves downstream so that the shock ceases to steepen. In this shock the velocity, density, and temperature undergo shock transition in a thin layer or subshock characterized by the viscous scale length. The magnetic field, responding only on the magnetic scale length, has a thicker shock transition. The argument is summarized in Figure 1.

The relative magnitudes of the downstream flow velocity and sound speed must be determined by solution of the Rankine-Hugoniot conditions as a function of the upstream Mach number and plasma conditions. For the perpendicular fast shock, Marshall (1955) showed that if the Mach number is sufficiently large, resistivity no longer provides the required dissipation, and a viscous subshock is formed.

3.2 Oblique Shocks

The possibility of a subshock exists whenever, after steepening to wavelengths comparable to one of the dissipation scale lengths, the downstream wave speed exceeds the downstream flow velocity. The allowed flow velocities are determined by the evolutionary conditions, and for comparison the appropriate hydromagnetic wave speeds are derived in section 2.0. In the next two sections Friedrich's diagrams are used to discuss separately possible subshock formation for resistive and thermal conduction fast and slow oblique shocks. A discussion of the effects of viscosity on the shock structure must await the analysis of section 4.0.

3.2.1 Resistive Shocks

Figure 2 is a sketch of a single quadrant of a Friedrich's diagram for $\beta_2 < 1$ and $\beta_2 > 1$. β is roughly the ratio of thermal to magnetic energy, and is defined here as $\beta = C_S^2/C_A^2$. First consider possible linear fast wave speeds and flow velocities behind a fast resistive shock propagating at an angle to the magnetic field for $\beta_2 < 1$. By the evolutionary conditions the flow speed U_2 can be less than the sound speed C_{S_2} if C_{S_2} exceeds the intermediate speed C_{I_2}. Therefore, from Figure 2 for the fast resistive shock subshock formation is possible over the range of angles $\pi/2 \geq \theta_2 \geq \cos^{-1}(C_{S_2}/C_{A_2})$. The corresponding limitation on θ_1 must, of course, be determined from the Rankine-Hugoniot conditions. For smaller angles, $U_2 > C_{I_2} > C_{S_2}$. Here the increase in the magnetic field strength across the fast shock is sufficient to prohibit further steepening, so that resistivity alone provides the necessary dissipation.

If $\beta_2 > 1$, there exists at all angles a possible U_2 such that $C_{I_2} \leq U_2 < C_{S_2}$. Therefore sufficiently strong $\beta_2 > 1$ fast shocks possess a subshock structure. Note that for $\beta_2 >> 1$, U_2 is less than C_{S_2} even for moderate strength shocks. Since for $\beta_2 >> 1$ the fast wave which steepens to form the shock is almost electrostatic, resistivity, which dissipates magnetic energy, has very little effect on the wave speed, and a subshock is to be expected.

The slow hydromagnetic wave speed is reduced to zero when $kr_m >> 1$. Since the fluid velocity behind the slow shock must be finite, no slow waves can reach the shock, and therefore the slow resistive shock does not form a subshock. Across the oblique slow shock the magnitude of the magnetic field decreases (Kantrowitz and Petschek, 1966), so that resistive dissipation is a required part of the shock structure.

3.2.2 Thermal Conductivity

Consider fast and slow shocks for which thermal conductivity is the primary dissipation process. The Friedrich's diagrams of the linear hydromagnetic wave speeds for $kr_t << 1$ (solid lines) and $kr_t >> 1$ (dashed lines) with $\beta_2 < 1$ and $\beta_2 > 1$ are presented in Figure 3. If $kr_t >> 1$, the linear wave speeds are defined with $\gamma = 1$ in the sound speed. By the previous considerations a subshock formation for thermal conduction fast and slow shocks is possible whenever the downstream flow velocity is less than the fast or slow linear speed, $\tilde{C}_{F_2}$ and $\tilde{C}_{SL_2}$, respectively. Such a subshock is the hydromagnetic analogue of the gas dynamic isothermal discontinuity. For fast shocks, $U_2 < \tilde{C}_{F_2}$ is possible at all downstream angles and β_2's except for $\beta_2 \sim 1$ when $\tilde{C}_{F_2} < C_{I_2}$ may occur at small angles. For slow shocks U_2 can be less than $\tilde{C}_{SL_2}$ for all downstream angles and β_2's. Note that for $\beta_2 << 1$, $C_{F_2} \sim \tilde{C}_{F_2}$, so that $U_2 < \tilde{C}_{F_2}$ only occurs for very weak fast shocks. Therefore for sufficiently strong fast and slow shocks thermal conduction alone is incapable of providing the required dissipation, and a subshock structure is expected.

The above discussion based on the linear hydromagnetic wave speeds and the evolutionary conditions about the downstream point only indicates the possibility that a subshock might be formed. An explicit determination requires solution of the Rankine-Hugoniot conditions and will not be discussed here. In the next section a method is developed to examine the effects of the various dissipation processes for both the upstream and downstream shock conditions.

4.0 Perturbation about the Shock Stationary Points

4.1 Introduction

To investigate the steady shock structure, equations 2.1 are assumed to be time independent in a frame of reference moving with the shock. A specific coordinate system is chosen such that the shock plane is perpendicular to the x axis and the magnetic field is contained in the $x - z$ plane; $x \to -\infty$ is upstream, $x \to +\infty$ is downstream. All quantities are assumed to vary only in the x direction. Equations 2.1 are then integrated once with respect to x to obtain

$$\rho U = \rho_1 U_1 \tag{4.1}$$

$$\rho U^2 + P + \frac{B_z^2}{8\pi} = \left(\frac{4}{3}\eta + \zeta\right)\frac{dU}{dx} + A_1 \tag{4.2}$$

$$\rho U V_z - \frac{B_x B_z}{4\pi} = \eta\,\frac{dV_z}{dx} - \frac{B_x B_{z_1}}{4\pi} \tag{4.3}$$

$$\rho_1\, U_1\left(\frac{U^2 + V_z^2}{2} + \frac{\gamma}{\gamma - 1}\,\frac{P}{\rho}\right) - \frac{B_z\left(V_z B_x - U B_z\right)}{4\pi} \tag{4.4}$$

$$= \frac{c^2}{(4\pi)^2\sigma}\,\frac{d}{dx}\left(\frac{B_z^2}{2}\right) + \left(\frac{4}{3}\eta + \zeta\right)\frac{d}{dx}\left(\frac{U^2}{2}\right) + \eta\,\frac{d}{dx}\left(\frac{V_z^2}{2}\right) + \kappa\,\frac{dT}{dx} + A_2$$

$$V_z B_x - U B_z + \frac{c^2}{4\pi\sigma}\,\frac{dB_z}{dx} = -U_1 B_{z_1} \tag{4.5}$$

U and V_z are the x and z flow velocities, respectively; A_1 and A_2 are constants of integration. The momentum equation for V_y and the B_y Ohm's law admit only the solution $V_y = B_y = 0$, in agreement with the co-planarity theorem. B_x is, of course, constant across the shock.

Ideally, to solve for the shock structure equations 4.1 – 4.5 are reduced to a single differential equation for one variable which describes the change in that quantity from the upstream stationary point to the downstream stationary point (Anderson, 1963). At the stationary points all gradients are reduced to zero and the plasma quantities satisfy the Rankine-Hugoniot conditions. The non-linearity of the above equations often renders obtaining a single general differential equation difficult.

Rather than struggle with non-linear differential equations, some of the effects of the various dissipation processes on the shock structure can be obtained by studying the response of equations 4.1 — 4.5. to a perturbation of the plasma parameters about the two stationary points. For a shock transition to occur, the upstream (downstream) point must be unstable (stable) to the perturbation in the direction of increasing x. The definition of stability in this paper is not equivalent to the definition employed in the study of shock stability with respect to hydromagnetic perturbations (Akhiezer et al., 1959; Germain, 1960; Anderson, 1963; Gardner and Kruskal, 1964). There the perturbed plasma parameters are restricted to satisfy the linear hydromagnetic dispersion relation and the Rankine-Hugoniot conditions. Here the perturbation is arbitrary. Finally, since only the linear response of equations 4.1 — 4.5 is desired, the assumed constancy of the dissipation coefficients is justified since, about the stationary points, changes in the dissipation coefficients produce terms of second order in the perturbation.

Following the previous format each dissipation process is considered separately. For completeness the rotational discontinuity is also included in the discussion.

4.2 Resistive Dissipation

Setting the coefficients of viscosity and thermal conductivity equal to zero, and performing a perturbation expansion of the plasma parameters about their value at a stationary point, e.g., $U = U_1 + \delta U$,

$\delta U/U_1 << 1$, equations 4.1 – 4.5 become a set of linear differential equations for the perturbed quantities. It is convenient to obtain the differential equation for the perturbed magnetic field, δB_z. After substituting 4.5 in 4.4, eliminating δP using the x-momentum equation 4.2, eliminating V_z and δV_z by 4.3, the energy equation 4.4 reduces to an equation involving only δU and δB_z. Using this equation and 4.3 in the Ohm's law 4.5, the following equation for δB_z is obtained

$$\left\{\left(\frac{B_x^2}{4\pi\rho} - U^2\right)\left(U^2 - \frac{\gamma P}{\rho}\right) - (\gamma - 1)\, B_z\left(\frac{B_x^2}{4\pi\rho}\, \frac{B_z - B_{z_1}}{4\pi\rho} + \frac{UU_1 B_{z_1}}{4\pi\rho} - \frac{\gamma}{\gamma - 1}\, \frac{U^2 B_z}{4\pi\rho}\right)\right\} \frac{\delta B_z}{U^2 - \frac{\gamma P}{\rho}} = - \frac{C^2 U}{4\pi\sigma}\, \frac{d\delta B_z}{dx} \tag{4.6}$$

All plasma quantities without specific subscripts are evaluated at either the upstream or downstream stationary point, and are, therefore, constant. To eliminate $U_1 B_{z_1}$ in 4.6, 4.5 and 4.3 are evaluated at a stationary point to obtain

$$U_1 B_{z_1} = UB_z - V_z B_x = UB_z - \frac{B_x^2\left(B_z - B_{z_1}\right)}{4\pi\rho U}$$

Substituting for $U_1 B_{z_1}$ and rewriting the equation in terms of the hydromagnetic fast, slow, and sound speeds, 4.6 becomes

$$\frac{C^2}{4\pi\sigma U}\, \frac{d\delta B_z}{dx} = \frac{\left(U^2 - C_F^2\right)\left(U^2 - C_{SL}^2\right)}{U^2\left(U^2 - C_S^2\right)}\, \delta B_z \tag{4.7}$$

The coefficient of the gradient is the resistive scale length defined with respect to flow velocity. The fast and slow speeds are defined by 2.6 without the dissipation terms where θ is now the angle between the x-axis and the magnetic field vector.

The solution of 4.7 is a simple exponential function of x. The sign of the coefficient of δB_z on the right hand side (RHS) determines whether, for increasing x, the perturbation grows or decays in space. Without loss of generality, each stationary point can be considered to occur at x = 0. The evolutionary conditions for the particular shock or discontinuity of interest fix the relationship between U and the hydromagnetic speeds at the upstream and downstream stationary points thus determining the sign of the RHS of 4.7. With the above restricted definition of stability, the effect of an arbitrary perturbation on a resistive shock and rotational discontinuity is investigated in the following sections. Since many of the conclusions have already been presented in section 3.0, the discussion below will stress new features.

4.2.1 Fast Resistive Shocks

The upstream flow velocity for a fast shock exceeds all hydromagnetic propagation speeds, so that the RHS of 4.7 is positive, and the upstream point is unstable. Therefore all fast shocks can be initiated by resistivity. For $\beta_1 \gg 1$ it seems curious that resistive dissipation leads to a shock transition since magnetic field changes are expected to be an inconsequential part of the shock structure. However, the magnetic field energy must always increase across a fast shock so that resistive dissipation, although weak, is important for the transition process.

About the downstream stationary point first note that θ_2 cannot equal zero for $\beta_2 < 1$ since the parallel fast shock is a complete switch-on shock for these conditions (Kantrowitz and Petschek, 1966). If $\beta_2 > 1$ and $\theta_2 = 0$, $C_{F_2} = C_{S_2}$ so that the denominator in 4.7 is canceled by the first term in the numberator. Since $U_2 \geq C_{A_2} = C_{SL_2}$, the downstream point is unstable. For all other angles and β's the downstream point is stable if $U_2 > C_{S_2}$ and unstable if $U_2 < C_{S_2}$. If $\beta_2 < 1$, the range of angles for which $U_2 < C_{S_2}$ can occur is $\pi/2 \geq \theta_2 \geq \cos^{-1}\left(\frac{C_{S_2}}{C_{A_2}}\right)$.

An unstable downsteam point implies that resistivity is incapable of preventing further shock steepening and establishing a complete shock transition. A viscous subshock, therefore, is necessary for a stable shock transition.

4.2.2 Rotational Discontinuity

From the evolutionary conditions for the rotational discontinuity the numerator of 4.7 for both the upstream and downstream conditions is negative. Note that if $\theta_1 = \theta_2 = 0$, the flow velocity equals the intermediate speed at either the upstream point if $\beta_1 < 1$ or the downstream point if $\beta_2 > 1$; hence these rotational discontinuities are of zero strength. The upstream point is stable (unstable) if $U_1 > C_{S_1}$, ($U_1 < C_{S_1}$). The downstream point is stable (unstable) if $U_2 > C_{S_2}$ ($U_2 < C_{S_2}$). Since for finite dissipation the flow velocity must decrease and the temperature increase across the discontinuity, if $U_1 > C_{S_1}$, so that the transition is initiated, U_2 cannot exceed C_{S_2} so that the downstream point must be unstable. Since $U_1 < C_{S_1}$, $U_2 > C_{S_2}$ is the only combination that might produce a well defined transition, there is no stable resistive rotational discontinuity.

4.2.3 Resistive Slow Shocks

First consider the upstream conditions $\theta_1 \neq 0$ and $\beta_1 < 1$. By the evolutionary conditions for slow shocks, the numerator of 4.7 is negative. Therefore if $U_1 > C_{S_1}$ ($U_1 < C_{S_1}$), the upstream point is stable (unstable). Note that these conditions include the complete switch-off shock. From Figure 2, U_1 can exceed C_{S_1} in the range of angles $0 \leq \theta_1 \leq \cos^{-1}\left(\frac{C_{S_1}}{C_{A_1}}\right)$; for larger angles $C_{S_1} > C_{I_1}$, and the upstream point is unstable. If $\theta_1 = 0$, the gas dynamic shock limit, the upsteam point is stable unless $U_1 = C_{A_1}$, the maximum strength slow shock, for which the RHS of 4.7 vanishes. Therefore if $0 \leq \theta_1 \leq \cos^{-1}\left(\frac{C_{S_1}}{C_{A_1}}\right)$, $\beta_1 < 1$, and $U_1 > C_{S_1}$, the slow shock transition cannot be initiated by resistive dissipation.

The interpretation of this result follows from considering the steepening of the slow wave. In this range of angles and $\beta_1 < 1$, the slow wave is primarily electrostatically polarized (Formisano and Kennel, 1969). In the shock formed by this wave the magnetic energy available for heating the plasma is small. (Across the slow shock the magnetic energy always decreases). Therefore if the $\beta_1 < 1$ slow shock is strong enough, $U_1 > C_{S_1}$, resistivity alone is incapable of initiating the shock steepening, and another dissipation mechanism is required. If the $\beta_1 < 1$ slow shock is weak, $U_1 < C_{S_1}$, magnetic dissipation is sufficient to slow the upstream fluid and initiate the shock. For $\pi/2 > \theta_1 \geq \cos^{-1}\left(\frac{C_{S_1}}{C_{A_1}}\right)$ the linear slow wave acquires a substantial polarization along the magnetic field. The resulting slow shock has sufficient magnetic energy available for dissipation so that resistivity starts the shock transition.

If $\beta_1 > 1$ and $\theta_1 \neq 0$, the RHS of 4.7 is positive, and resistivity initiates the slow shock. If $\theta_1 = 0$, $\beta_1 > 1$, $U_1 = C_{A_1} = C_{SL_1}$, and the shock is of zero strength. For $\beta_1 > 1$ the linear slow wave is primarily electromagnetically polarized. Therefore the dissipation mechanism in the slow shock formed by the steepening of this wave should be primarily resistive.

The downstream point of resistive slow shock is stable to perturbations for all θ_2 and β_2 in agreement with the conclusion of section 3.2.1. Therefore if resistivity starts the slow shock, resistive dissipation alone is capable of providing a complete shock transition.

4.3 Thermal Conductivity

If resistivity and viscosity contribute negligibly to the dissipation, equations 4.1 – 4.5 describe the transition for a thermal conduction shock. As above, the equations are perturbed about a stationary point, and, after eliminating the other perturbed quantities in terms of δU, the following equation is obtained

$$\frac{(\gamma - 1)\kappa}{\rho U}\frac{d\delta U}{dx} = \frac{\left(U^2 - C_F^2\right)\left(U^2 - C_{SL}^2\right)}{\left(U^2 - \tilde{C}_F^2\right)\left(U^2 - \tilde{C}_{SL}^2\right)}\,\delta U \tag{4.8}$$

On the left hand side the coefficient of the gradient is proportional to thermal conduction scale length defined with respect to the flow velocity. As before $\tilde{C}_F$, $\tilde{C}_{SL}$ are the fast and slow speeds with γ set equal to unity in the sound speed. Since many of the results were presented in section 3.2.2, the discussion here will be brief.

4.3.1 Fast Thermal Conduction Shocks

From 4.8 the upstream point is unstable for all angles and β's so that the fast shock can be initiated by thermal dissipation. If $U_2 > \tilde{C}_{F_2}\left(U_2 < \tilde{C}_{F_2}\right)$, the downstream point is stable (unstable) to an arbitrary perturbation. Therefore, since $\tilde{C}_F$ differs only slightly from C_F, only for weak shocks can thermal conduction provide all the required dissipation. For stronger shocks, either a resistive or viscous subshock is formed.

4.3.2 Rotational Discontinuity

Using the evolutionary conditions, from 4.8 the upstream point of the rotational discontinuity is stable (unstable) if $U_1 > \tilde{C}_{F_1}$ $\left(U_1 < \tilde{C}_{F_1}\right)$. The downstream point is stable (unstable) if $U_2 > \tilde{C}_{F_2}$ $\left(U_2 < \tilde{C}_{F_2}\right)$. Since the pressure must increase and the flow velocity decrease across the discontinuity, the pair $U_1 < \tilde{C}_{F_1}$ and $U_2 > \tilde{C}_{F_2}$, which is the only stable transition, cannot occur. Therefore the rotational discontinuity is disallowed for thermal dissipation.

4.3.3 Slow Thermal Conduction Shocks

The upstream point is unstable for all angles and β's except the small range of $\beta_1 > 1$ such that $C_{I_1} > U_1 > \tilde{C}_{F_1}$ which is stable. Hence thermal conduction can initiate most slow shock transitions. The downstream point is stable (unstable) if $U_2 > \tilde{C}_{SL_2}$ $\left(U_2 < \tilde{C}_{SL_2}\right)$. Therefore thermal conductivity provides all the necessary dissipation only for weak slow shocks.

4.4 Viscosity

Dropping the resistive and thermal conduction terms, and eliminating V_z, δV_z, and δP, equations 4.1 — 4.5 reduce to the coupled equations

$$\rho U\left(U\delta B_z + B_z\delta U\right) - \frac{B_x^2}{4\pi}\,\delta B_z = \eta\left(U\,\frac{d\delta B_z}{dx} + B_z\,\frac{d\delta U}{dx}\right) \tag{4.9}$$

$$\left(\frac{4}{3}\,\eta + \zeta\right)\frac{d\delta U}{dx} = \rho\left(U - \frac{\gamma P}{\rho U}\right)\delta U + \frac{B_z}{4\pi}\,\delta B_z \tag{4.10}$$

For a parallel propagating shock, $B_z = 0$, 4.9 and 4.10 decouple and are solvable separately for δB_z and δU. For oblique propagation 4.9 and 4.10 combine into a single second order differential equation for δU. These two cases will be considered separately.

4.4.1 Parallel Shocks

If $B_z = 0$ both upstream and downstream, the solutions of 4.9 and 4.10 are

$$\delta U = \delta U_0\,\exp\left(\frac{\rho U}{\frac{4}{3}\,\eta + \zeta}\;\frac{U^2 - C_S^2}{U^2}\,x\right) \tag{4.11}$$

$$\delta B_z = \delta B_{z0}\,\exp\left(\frac{\rho U}{\eta}\;\frac{U^2 - C_I^2}{U^2}\,x\right) \tag{4.12}$$

where δU_0 and δB_{z0} are initial perturbations.

First consider the fast shock for $\beta_1 > 1$, $\beta_2 > 1$. From 4.11 since $U_1 > C_{S_1}$, and $U_2 < C_{S_2}$ by the evolutionary conditions, the upstream (downstream) velocity perturbation is unstable (stable), indicating a well-behaved shock transition. Also the upstream point is unstable if $\beta_1 < 1$. For $\beta_2 < 1$ the fast shock is a switch-on shock, and $B_{z2} \neq 0$. Since the flow velocity always exceeds the intermediate speed, 4.12 indicates that magnetic perturbation grows at both the upstream and downstream points. Therefore the fast shock does not remain parallel. An interpretation of these results will be delayed until after the oblique case has been discussed.

The parallel slow shock for $\beta_1 > 1$ and the parallel rotational discontinuity for $\beta_1 < 1$ are of zero strength since $U_1 = C_{I1}$. The $\beta_1 > 1$ rotational discontinuity is stable to velocity perturbations upstream and downstream and is unstable to magnetic perturbations upstream. Hence the parallel rotational discontinuity cannot be started.

The parallel slow shock for $\beta_1 < 1$ is unstable (stable) to velocity perturbations upstream (downstream). Since the flow velocity never exceeds the intermediate speed, the solution of 4.12 is $\delta B_z = 0$ upstream and downstream. Therefore the $\beta_1 < 1$ parallel slow shock has no magnetic changes across it and is the plasma analogue of the gas dynamic shock. For this shock viscosity alone provides a well defined transition.

4.4.2 Oblique Shocks

If $B_z \neq 0$, by solving 4.10 for δB_z and substituting into 4.9, the following second order differential equation for δU is obtained

$$\frac{\eta\left(\frac{4}{3}\eta + \zeta\right)}{\rho^2}\,\frac{d^2\delta U}{dx^2} - \left(\left(U^2 - C_S^2 - C_A^2\sin^2\theta\right)\frac{\eta}{\rho U} + \left(U^2 - C_I^2\right)\frac{\frac{4}{3}\eta + \zeta}{\rho U}\right)\frac{d\delta U}{dx}$$

$$+ \left(U^2 - C_F^2\right)\left(U^2 - C_{SL}^2\right)\frac{\delta U}{U^2} = 0 \qquad (4.13)$$

To solve 4.13 δU is assumed to vary as $\exp(\lambda x)$, and the resulting quadratic equation in λ has as solutions

$$\frac{\eta\left(\frac{4}{3}\eta + \zeta\right)U}{\rho}\,\lambda_{\pm} = \left(\frac{4}{3}\eta + \zeta\right)\left(U^2 - C_I^2\right) + \eta\left(U^2 - C_S^2 - C_A^2\sin^2\theta\right)$$

$$\pm\left[\left(\eta\left(U^2 - C_S^2 - C_A^2\sin^2\theta\right) - \left(\frac{4}{3}\eta + \zeta\right)\left(U^2 - C_I^2\right)\right)^2 \qquad (4.14)$$

$$+ 4\,\eta\left(\frac{4}{3}\eta + \zeta\right)C_I^2 C_A^2\sin^2\theta\right]^{1/2}$$

The $\pm$ subscripts on λ refer to the positive and negative square roots, respectively. Note that $\lambda_{\pm}$ is real.

If the limit $B_z = 0$, i.e., $\theta = 0$, is taken in 4.14, λ_+ corresponds to the solution 4.11 and λ_- to 4.12. Hence the solutions for δU and δB_z which are continuous at $\theta = 0$ are

$$\delta U = \delta U_0\,\exp\left(\lambda_+ x\right) \qquad (4.15)$$

$$\delta B_z = \delta B_{z0}\,\exp\left(\lambda_- x\right) \qquad (4.16)$$

Two further expressions are useful in discussing these solutions. From the properties of quadratic equations and 4.13, the product of $\lambda_+ \lambda_-$ is given by

$$\lambda_+\lambda_- = \frac{\rho^2}{\eta\left(\frac{4}{3}\eta + \zeta\right)} \frac{\left(U^2 - C_F^2\right)\left(U^2 - C_{SL}^2\right)}{U^2} \tag{4.17}$$

In the range of angles $\theta << 1$ and $\theta \sim \frac{\pi}{2}$, the square root in 4.14 can be expanded to obtain the approximate solutions

$$\frac{\left(\frac{4}{3}\eta + \zeta\right)U}{\rho} \lambda_+ \approx \left(U^2 - C_S^2 - C_A^2 \sin^2\theta\right) + \\ + \frac{\left(\frac{4}{3}\eta + \zeta\right) C_I^2 C_A^2 \sin^2\theta}{\left[\eta\left(U^2 - C_S^2 - C_A^2 \sin^2\theta\right) - \left(\frac{4}{3}\eta + \zeta\right)\left(U^2 - C_I^2\right)\right]} \tag{4.18}$$

$$\frac{\eta U}{\rho} \lambda_- \approx U^2 - C_I^2 - \frac{\eta\, C_I^2 C_A^2 \sin^2\theta}{\left[\eta\left(U^2 - C_S^2 - C_A^2 \sin^2\theta\right) - \left(\frac{4}{3}\eta + \zeta\right)\left(U^2 - C_I^2\right)\right]} \tag{4.19}$$

For simplicity the stability discussion below will concentrate on near parallel and perpendicular propagation. Extension to arbitrary oblique angles is accomplished by 4.17.

4.4.3 Fast Shocks

About the upstream point $\lambda_+ > 0$ and $\lambda_- > 0$ for both $\theta_1 << 1$ and $\theta_1 \sim \frac{\pi}{2}$. Since from 4.17 $\lambda_+\lambda_- > 0$, it is likely that viscosity is always capable of starting the fast shock transition. About the downstream point 4.17 yields $\lambda_+\lambda_- < 0$. From 4.18 and 4.19 $\lambda_+ \gtrless 0$ and $\lambda_- \lessgtr 0$ if $U_2^2 \gtrless C_{S_2}^2 + C_{A_2}^2 \sin^2\theta_2$. Therefore the downstream point for the fast shock is unstable to either velocity or magnetic perturbations, and viscosity alone is incapable of providing a complete fast shock transition. Since the magnetic field increases across all fast shocks, resistive dissipation must be part of the shock structure even though the majority of the dissipation is accomplished by viscosity. Therefore the structure of fast shocks, for which resistivity alone is insufficient, consists of a broad resistive region for the magnetic field change and a viscous subshock.

4.4.4 Rotational Discontinuity

From 4.17 $\lambda_+\lambda_- < 0$ for both upstream and downstream points. Therefore the viscous rotational discontinuity is always unstable to an arbitrary perturbation.

4.4.5 Slow Shocks

The upstream point for the viscous slow shock has $\lambda_+\lambda_- < 0$. Across the parallel $\beta_1 < 1$ slow shock the magnetic field is unchanged so that $\lambda_- < 0$ simply implies that $\delta B_z = 0$. For oblique propagation, however, the magnetic field decreases across the slow shock so that $\lambda_+\lambda_- < 0$ indicates that viscosity alone is insufficient for a complete shock transition.

First consider the slow shock for $\beta_1 < 1$ and $\theta_1 << 1$. From 4.18 and 4.19 $\lambda_+ \gtrless 0$ and $\lambda_- \lessgtr 0$ if $U_1^2 \gtrless C_{S_1}^2 + C_{A_1}^2 \sin^2\theta_1$. If the shock is strong, $U_1 > C_{S_1}$, viscosity starts the velocity but not the magnetic transition. The results of section 4.2.3 indicated that for these parameters resistivity alone could not start the slow shock. Therefore the shock structure probably contains an upstream viscous layer across which the velocity changes; resistive dissipation then decreases the magnetic field. Weaker slow shocks, $U_1 < C_{S_1}$ are not started by viscosity but are by resistivity, and therefore probably possess only a resistive structure.

For $\theta_1 << 1$, $\beta_1 > 1$, 4.18 and 4.19 yield $\lambda_+ < 0$ and $\lambda_- > 0$ so that viscosity does not start the velocity transition. Also if $\theta_1 \sim \frac{\pi}{2}$, $\lambda_+ < 0$ and $\lambda_- > 0$ for all β_1. In these two cases the magnetic field experiences a large change across the slow shock, and therefore, considering the results of section 4.2.3, resistivity probably dominates the shock structure.

From 4.17 the downstream point of the slow shock has $\lambda_+\lambda_- > 0$. Both $\theta_2 << 1$ and $\theta_2 \sim \frac{\pi}{2}$ have $\lambda_+ < 0$ and $\lambda_- < 0$ so it is likely that the downstream point is always stable. Recall from section 4.2.3 that the downstream point is also stable for resistive slow shocks.

5.0 Discussion

The equations of hydromagnetics including dissipation describe the shock transition between the upstream and downstream stationary points at which the plasma quantities obey the Rankine-Hugoniot jump conditions. The effect of each type of dissipation mechanism on the shock structure was determined by solving for the linear response to an arbitrary perturbation about the two stationary points. The evolutionary conditions prescribed whether the perturbations grew or decayed in space. A well defined shock transition for a particular form of dissipation is one in which the upstream point is unstable and the downstream point is stable to perturbations. The conclusions of the analysis are summarized below:

1. The upstream point of the fast resistive shock is always unstable, indicating that resistivity starts the shock steepening. The downstream point in the range of angles $\frac{\pi}{2} \geq \theta_2 \geq \cos^{-1}\left(\frac{C_{S2}}{C_{A2}}\right)$ for $\beta_2 < 1$ and at all angles for $\beta_2 > 1$ is stable if $U_2 > C_{S_2}$ and unstable if $U_2 < C_{S_2}$; for $\beta_2 < 1$ and $0 \leq \theta_2 \leq \cos^{-1}\left(\frac{C_{S2}}{C_{A2}}\right)$ the downstream point is stable. If the downstream point is stable (unstable), resistivity alone is (not) capable of prohibiting further shock steepening and providing the required dissipation. If unstable, a subshock is necessary for a complete transition.

2. For $\beta_1 < 1$ and $0 \leq \theta_1 \leq \cos^{-1}\left(\frac{C_{S1}}{C_{A1}}\right)$ the upstream point of the resistive slow shock is stable if $U_1 > C_{S_1}$. The linear slow waves from which this shock steepens are primarily electrostatically polarized. Since the resultant shock has little magnetic energy available for dissipation, resistivity alone cannot start the shock steepening and

another dissipation process is required. For larger angles and all $\beta_2 > 1$ shocks, the upstream point is unstable — a reasonable conclusion since the shock is formed by waves that are primarily electromagnetic. The downstream point is stable for all β_2 and θ_2 so that resistivity can complete the slow shock transition.

3. Thermal conduction shocks possess unstable downstream points whenever for fast shocks $U_2 < \tilde{C}_{F_2}$ and for slow shocks $U_2 < \tilde{C}_{SL_2}$. Since $\tilde{C}_F$ and $\tilde{C}_{SL}$ differ little from C_F and C_{SL}, respectively, thermal conductivity provides the necessary dissipation only in the case of weak shocks. Stronger shocks require either a resistive or viscous subshock.

4. Viscosity slows all hydromagnetic wave speeds to zero, and therefore is the strongest form of dissipation. The upstream point of the viscous fast shock is always unstable. The downstream point is unstable to either velocity or magnetic perturbations indicating that viscosity alone cannot complete the shock transition. Some resistive dissipation is necessary to increase the magnetic field across the shock. Therefore the strong fast shock structure consists of both a resistive layer and a viscous subshock.

5. The parallel $\beta_1 < 1$ slow shock, having steepened from a purely electrostatic linear wave, requires no magnetic dissipation so that viscosity alone is sufficient for the shock transition. The oblique slow shock cannot be completely started by viscosity since either the velocity or the magnetic field perturbation is stable. For $\beta_1 < 1$ and $\theta_1 << 1$ strong shocks, $U_1 > C_{S_1}$, have both viscous and resistive dissipation whereas weak shocks, $U_1 < C_{S_1}$, probably are resistive. Very oblique shocks for $\beta_1 < 1$ and all $\beta_1 > 1$ shocks, which have substantial magnetic changes across them, require resistivity to initiate the shock steepening. The downstream point of the viscous slow shock is stable to perturbations.

6. The rotational discontinuity is disallowed for resistive, thermal conduction, and viscous dissipation since no unstable-upstream, stable-downstream combination exists.

In this paper no attempt has been made to determine at what Mach number a particular dissipation process fails to provide a complete shock transition. The relevant criteria developed here combined with the Rankine-Hugoniot conditions would provide the solution to this problem. When more than one dissipation process is required, details of the shock structure must, of course, be determined by solving the non-linear hydromagnetic differential equations.

Finally a comment on the applicability of the above results to collisionless shocks is perhaps warranted. In a collisionless shock dissipation mechanisms other than those considered here, such as equalization of electron-ion temperature differences, relaxation of pressure anisotropy, wave-wave and wave-particle interactions, might contribute to the shock structure. Each in turn must be examined as to whether or not the required dissipation can be supplied. Some conclusions of the hydromagnetic theory should, however, be applicable. The anomalous dissipation for strong fast shocks must include both viscous and resistive interactions. The dissipation for very oblique slow shocks is expected to have the form of resistivity. Almost parallel strong slow shocks should require both an anomalous viscosity, which probably provides most of the dissipation, and some anomalous resistivity. Dissipation by anomalous thermal conduction is sufficient only for very weak fast and slow shocks.

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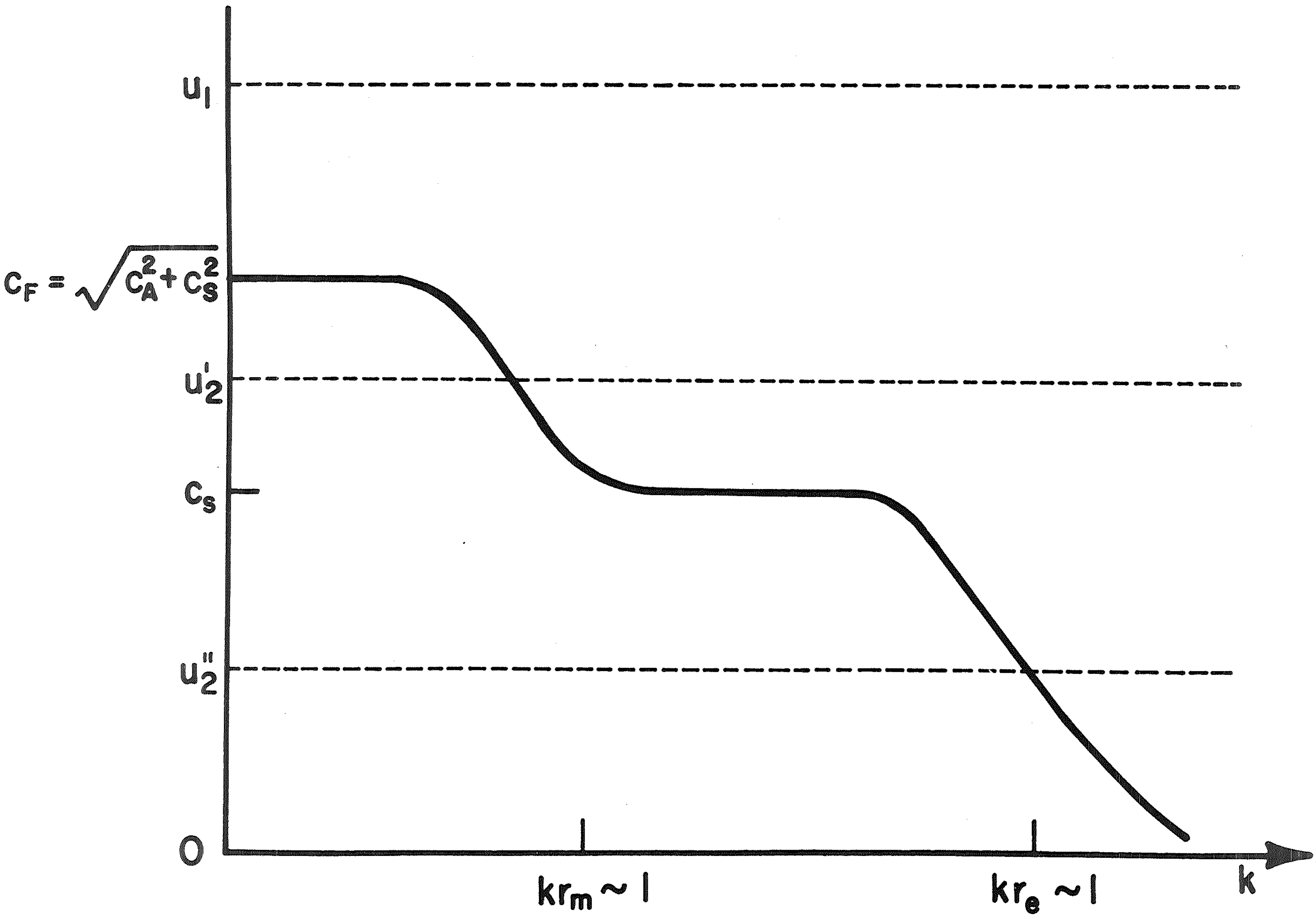
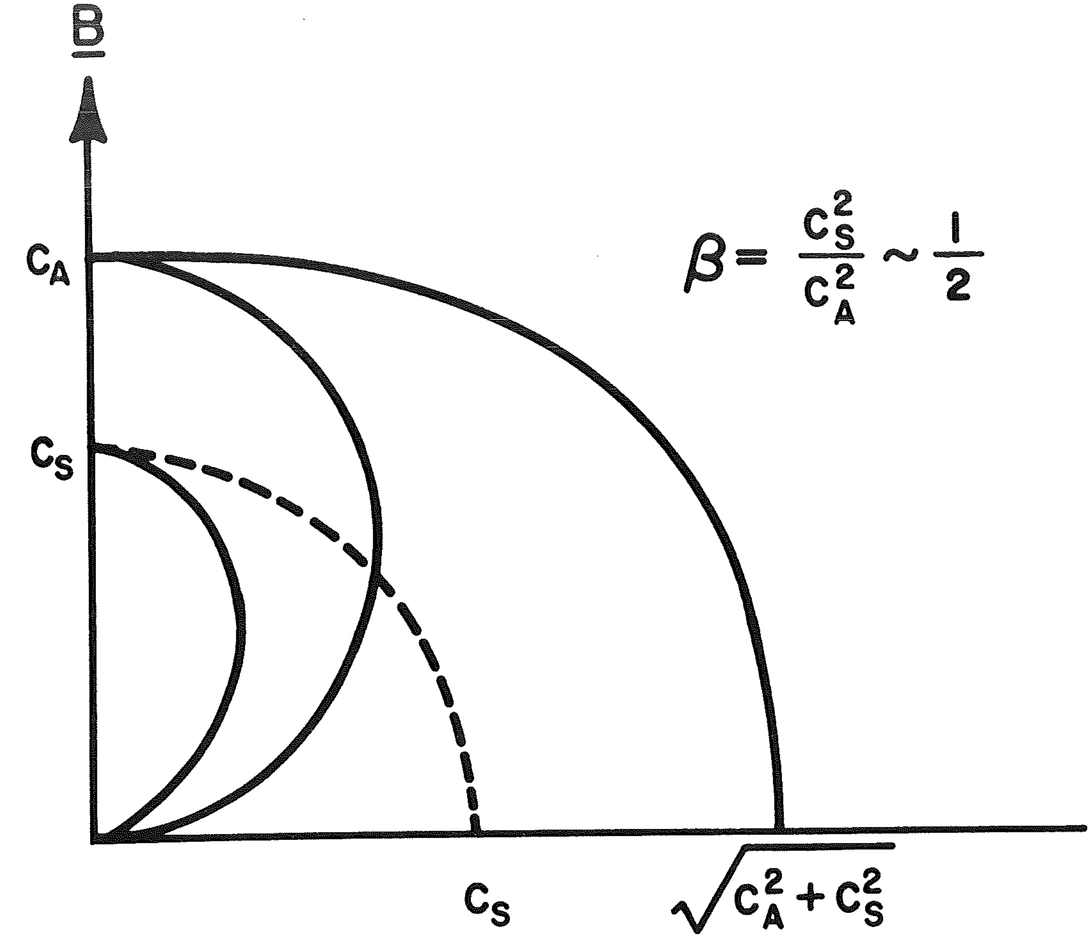
FIGURE CAPTIONS

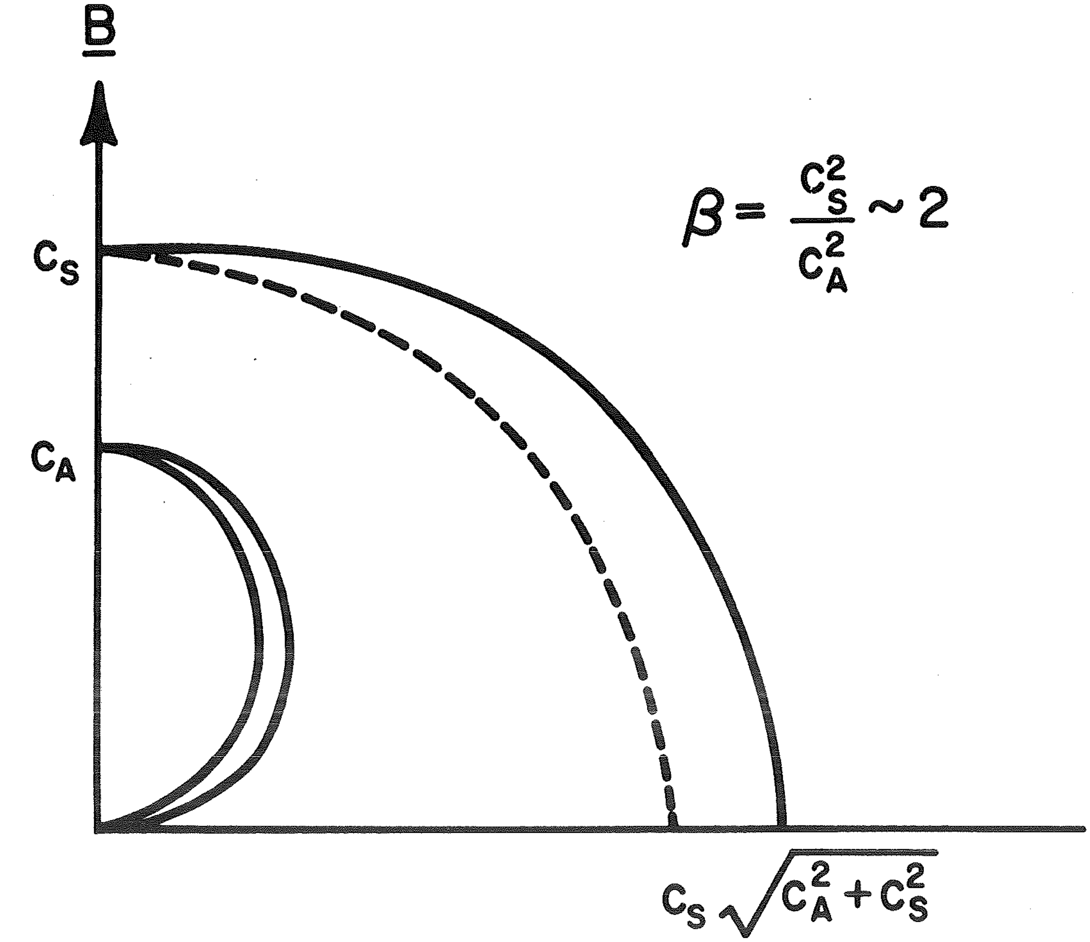
Figure 1. The phase velocity of the perpendicular fast wave vs the wave vector k is sketched: when $kr_m >> 1$, C_F is reduced to C_S; when $kr_e >> 1$, C_F is reduced to zero. For comparison a typical upstream flow velocity $U_1 > C_F$, and two possible downstream flow velocities $C_F > U_2' > C_S > U_2''$ are included. If $U_2 > C_S$, waves propagating toward the shock on the downstream side are convected away by the fluid; shock steepening ceases and a steady shock of thickness $\sim r_m$ is formed. For strong shocks such that $U_2'' < C_S$ downstream waves continue to reach the shock causing further steepening until the viscous scale length, r_e, is reached. Since downstream waves have $C_F \to 0$ when $kr_e >> 1$ they are convected away from the shock; shock steepening ceases and a steady shock with a viscous subshock of thickness $\sim r_e$ and a more gradual magnetic shock of thickness $\sim r_m$ is formed.

Figure 2. A quarter quadrant of a Friedrich's diagram for $\beta \sim 1/2$ and $\beta \sim 2$ (solid lines) including the sound speed (dashed line) is sketched.

Figure 3. A quarter quadrant of a Friedrich's diagram for $\beta \sim 1/2$ and $\beta \sim 2$ is sketched (solid lines). Also shown (dashed lines) are the fast $\tilde{C}_F$ and slow $\tilde{C}_{SL}$ speeds with $\gamma = 1$ appropriate for isothermal propagation when $kr_t >> 1$.


u_1
$C_F = \sqrt{C_A^2 + C_S^2}$
u_2'
C_S
u_2''
0
$kr_m \sim 1$
$kr_e \sim 1$
k


B
C_A
C_S
$\beta = \frac{C_S^2}{C_A^2} \sim \frac{1}{2}$
C_S
$\sqrt{C_A^2 + C_S^2}$


B
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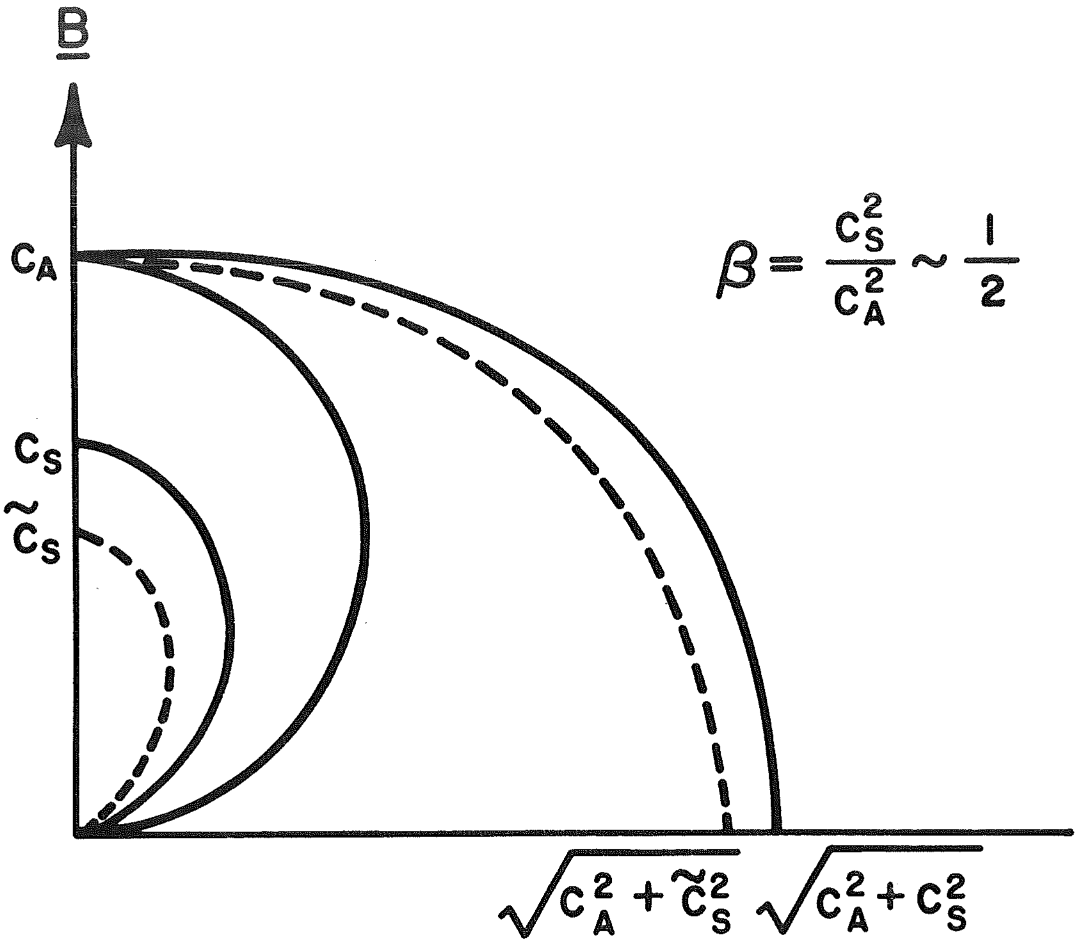
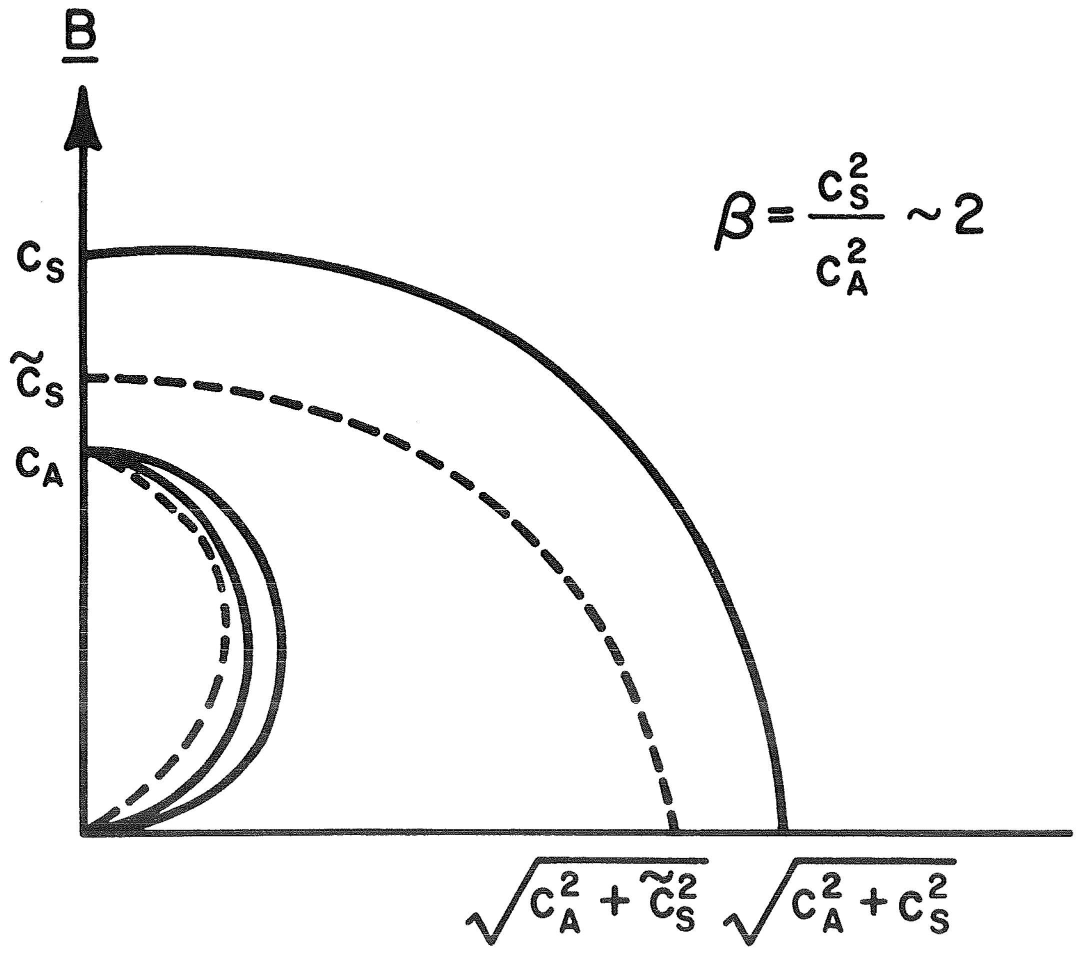

B
$\beta = \frac{C_S^2}{C_A^2} \sim \frac{1}{2}$
C_A
C_S
$\tilde{C}_S$
$\sqrt{C_A^2 + \tilde{C}_S^2}$
$\sqrt{C_A^2 + C_S^2}$


B
$\beta = \frac{C_S^2}{C_A^2} \sim 2$
C_S
$\tilde{C}_S$
C_A
$\sqrt{C_A^2 + \tilde{C}_S^2}$
$\sqrt{C_A^2 + C_S^2}$